Dice Activities for Algebraic Thinking

Foreword

Successful math students use the concepts of algebra —patterns, relationships, functions, and symbolic representations—in constructing solutions to mathematical problems. The activities in ***Dice Activities for Algebraic Thinking*** were created to engage students in developing fluency with the mathematical concepts of square number, square root, prime number, factorials, summation, integers, and exponential notation. The activities are designed to empower students to analyze, represent, and make generalizations about patterns in all aspects of math and to address mathematical problems and challenges with curiosity and confidence. ***Dice Activities for Algebraic Thinking*** precedes formal work in algebra in which students employ deductive reasoning and step-by-step procedures to balance equations. The book provides opportunities to employ inductive/intuitive cognitive strategies to solve for *n*, thus deepening students' understanding of algebra and providing a foundation for further study.

Dice Activities for Algebraic Thinking addresses the Common Core State Standards of Number and Operations and Algebra. The book also addresses the Mathematical Practices of reasoning and problem solving. The Common Core Standards are the framework for over 47 states and the District of Columbia; thus, these activities are easily integrated into a scope and sequence whenever the topic is addressed. In many instances the teacher will want to replace the activities in the school-based text with those found in this book, as they are challenging and thus more apt to provide long-term mastery; they may also develop an interest in and curiosity about math.

The authors currently use ***Dice Activities for Algebraic Thinking*** as part of their curriculum to train elementary school teachers in how to teach mathematics. The activities have been field-tested in public, private, and charter schools, and require only the use of dice, a commonly available manipulative. They provide an opportunity for students to play with big mathematical ideas without paper-and-pencil drill. The activities are engaging, generate collaborative competition, and provide an immediate use and reason for learning these mathematic concepts.

Our work is continually expanding, and we welcome any suggestions for modification of these activities that will lead to greater mathematical understanding on the part of our students. Please submit any suggestions to: DiceAuthors@didax.com.

— Chet Delani and Mary Saltus

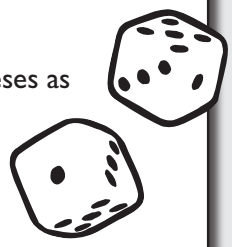
Contents

Notes to Teachers

This book was designed by the authors to assist students in constructing their own meanings and strategies for understanding abstract algebraic ideas. As with all books in the *Dice Activities* series, it offers a clear alternative to the preponderance of print that emphasizes rote learning and tedious practice drills without meaning and motivation. By employing interesting activities and fostering the attitude that math is enjoyable, we believe that students will increase their knowledge base and understanding of mathematical concepts and relationships.

In essence, ***Dice Activities for Algebraic Thinking*** is intended to:

- Construct a bridge for students to transition from thinking arithmetically to thinking algebraically
- Assist students in the use of letters to represent unknowns and variables
- Apply deductive and inductive reasoning in solving for *n*
- Facilitate the representation of the operation of multiplication by a "•" or parentheses as appropriate
- Generate rules to describe number patterns
- Increase and deepen an understanding of equivalency



Using a calculator is recommended for many of the activities because it puts the focus on developing algebraic thinking rather than computational efficiency. Look for the calculator icon to denote those activities for which a calculator is recommended.

Algebra as a Balancing Act

Algebra is about finding an unknown. It is a branch of mathematics that uses letters and symbols to represent the unknown. Algebra uses math statements to describe relationships between patterns, sequences, and functions. But algebraic thinking is not just arithmetic with letters. It requires thinking logically about number. ***Dice Activities for Algebraic Thinking*** specifically addresses the algebraic concepts of:

Using symbols:

- A dot • replaces the arithmetic multiplication symbol ×
- $\sqrt{\ }$ represents the square root of a number
- ! represents the factorial of a number
- ∑ represents the summation of a number
- n^a represents a number to an exponential power

Using the letter *n* to:

- Represent an unknown
- Identify the variable in a mathematical statement describing a sequence or pattern

Thinking logically about numbers:

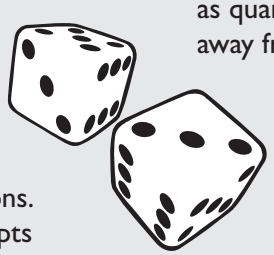
- Generating equalities from a random set of numbers
- Solving for *n*
- Writing rules for patterns, sequences, and functions
- Writing and solving simultaneous equations

Notes to Teachers (cont.)

Part I – Dice Activities

Part I begins with a **balancing act,** the basis of solving algebra problems. Students toss 4 dice. Using what they know about how numbers work, students manipulate the 4 tossed numbers to create an equality statement such that when the 4 numbers are placed on a scale, the scale balances. Generating equalities or equations from tossed dice sets the scene for much of the game aspect in Part 1. Students gain a deeper understanding of how numbers work by using not only +, –, •, and ÷ but also exponents, square roots, factorials, summations, and decimals, to arrive at an equality or equation.

The ***Hundred Chart*** activities employ two teams of two students per team and use a random set of numbers from dice tosses to generate equations. By recording equations, students have numerous opportunities to use a dot or parentheses to represent multiplication. Each chart focuses on a specific math concept—for example, multiples, factors, prime numbers, square numbers, and order of operations. Students are encouraged to use concepts beyond the four operations in creating equations, such as !, ∑, exponents, square roots, and decimals. The team that covers the most numbers on the chart wins.



The ***Integer*** activities are an engaging way to review the algorithms of adding, subtracting, and multiplying positive and negative numbers.

To play a ***Chart Activity*** (Four in a Row, Cross Over, or Tic-Tac-Toe), students first determine the value of *n* in each of the equations in the 25 boxes. Students encounter the unknown, *n*, on either side of the equal sign, developing the understanding of "equal" as quantitative sameness. The focus thus moves away from the left-to-right direction in solving equations.

The first activity in ***Writing Rules*** is a scripted lesson guiding teachers to elicit from students rules to describe patterns. A rule can be a statement or an algebraic expression. Several activities follow in which students practice writing rules to describe patterns and/or creating their own patterns to share with the class. These scripted lessons prepare the students to write rules for the ***Table Completion, Sequences,*** and ***Function Completion*** activities.

Part II – Tile Activities

The ***Balance*** activities also begin with a **balancing act.** Using the balance scale, students manipulate color tiles to solve one-step, two-step, and three-step **simultaneous equations.** As in the Chart activities, the tile problems develop the understanding of "equal" as quantitative sameness and move the focus away from the left-to-right direction in solving equations. Attention is on the equation as an entity. Students find solutions by removing tiles from both sides of the scale to maintain balance and/or substituting equivalent tiles to facilitate solving the equation. Students record their steps in solving each problem.

The ***Teacher Inquiry Scripts*** are four scripted lessons that demonstrate how to guide students to represent the tile problems as simple one-step, two-step, and simultaneous equations. Students refer to their recorded steps for solving each tile problem and represent the steps in algebraic form.

The ***Application*** problems apply algebraic thinking to solving problems. Students working in groups use tiles to construct the patterns the problems present and then record their data on a chart. From the concrete model and chart data, students write a rule or algebraic statement.

Directions for Scale-Toss Activities

Objectives

- Generate equalities using a random set of numbers.
- Practice use of the symbol • or parentheses to denote multiplication.

How to Play

- Toss 4 dice. Place all 4 dice on the scale so that it balances.
- Using all 4 numbers, record as many equalities as you can that keep the scales balanced.
- If possible, try to use at least one example each of Σ, $\sqrt{\ }$, !, n^2, n^3, decimals, and negative numbers.
- Use a • to represent multiplication instead of an ×.
- Record all your attempts below each scale, even though you may not find a solution. It will be a good record of your knowledge and strategies.
- If you cannot fill all nine scales on one toss, toss the dice again. Record the equalities from the second toss in a different-colored pen or pencil.
- See if you can complete the chart with just 3 tosses.

Example:

Dice Toss = 5, 5, 6, 4

–6 + 5	–5 + 4
Δ	
(5 • 5) – 4	Σ6
Δ	
5! ÷ 4	6 • 5
Δ	
0.5(6)	$5 - \sqrt{4}$
Δ	

Variation

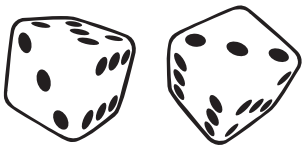
- Use 5, 6, 7, or more dice to generate equalities.

GETTING STARTED

List the square numbers, cubed numbers, summations, and factorials to help in creating equalities.

- Square numbers 1 through 12: _____
- Cubed numbers 1 through 5: _____
- Summations (Σ) 1 through 12: _____
- Factorials (!) 1 through 6: _____

Scale-Toss Chart



Δ	Δ	Δ
Δ	Δ	Δ
Δ	Δ	Δ

Recording Chart for Hundred Chart Activities

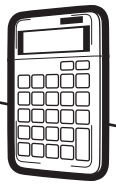
Make a list of:

- Numbers 1 through 12 squared: _____
- Numbers 1 through 5 cubed: _____
- Summations of numbers 1 through 12: _____
- Factorials of numbers 1 through 6: _____

Dice Tossed	Equation

Multiple Hundred Chart

- *Each team tosses a die. Higher number goes first.*
- *Each team chooses a color token.*



How to Play

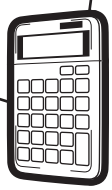
- Circle a number for this activity: 2, 3, 4, 5, 6, 7, 8, 9, or 10.
- Toss 4 dice. Using all 4 numbers tossed, write an equation that equals a multiple of the circled number. (Example: Toss 5, 4, 2, 2. Write the equation $(5^2 - 4) \times 2 = 42$ and place a token on 42.)
- Before you place a colored token on the number, the opposing team must agree to your solution. Keep a record of your equations on your team's recording chart.
- After 10 tosses, each team tallies the numbers under their tokens. The team with the higher score wins.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Not-a-Multiple Hundred Chart

How to Play

- *Each team tosses a die. Higher number goes first.*
- *Each team chooses a color token.*



- Circle a number for this activity: 2, 3, 4, 5, 6, 7, 8, 9, or 10.
- Toss 4 dice. Using all 4 numbers tossed, write an equation that is not a multiple of the circled number. (Example: The circled number is 2 and the numbers tossed are 5, 4, 2, 2. Write the equation $5^2 + (4 \cdot 2) = 33$ and place a token on 33, an odd number and not a multiple of 2.)
- Before you place a colored token on the number, the opposing team must agree to your solution. Keep a record of the equations you have written on your team's recording chart.
- After 10 tosses, each team tallies the numbers under their tokens. The team with the higher score wins.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Directions for Solve for *n* Chart Activities – Square Off, Cross Over, and Four in a Row

Objectives

- Develop a working knowledge of the mathematical concepts of exponents, square roots, factorials, summations, negative numbers, and fractions.
- Recognize the symbol • and parentheses as other representations for multiplication.
- Analyze the opponent's possible moves to develop a strategy to block the opponent.
- Identify the role of luck versus that of skill in an activity using dice.
- Develop communication and cooperation skills by working in teams of two students.

Introduce the ***Square Off, Cross Over,*** or ***Four in a Row*** activities by demonstrating on an interactive whiteboard or overhead and playing against the class. Teams with two students on a team are suggested. Teams give students an opportunity to discuss moves and strategies and provide a check on correct computation.

Materials

- Chart
- Dice
- Tokens

How to Play

- **Before** play begins, teams **work together** to solve for n on the chart.
- Each team tosses a die. The team with the higher number goes first.
- Team A tosses a die and finds an equation on the chart for which the tossed number is the solution for n. If the solution already has a token on it, the team loses a turn.
- Team A places a token on the equation. If Team B challenges the solution and the solution is incorrect, Team A's **turn is over.** If Team B challenges Team A's solution and the solution is correct, Team B **loses its next turn.**

(See next page for directions for playing ***Square Off, Four in a Row,*** and ***Cross Over.***)

Suggestion

- With more difficult levels of play, or if students are struggling, suggest the use of a calculator.

Discussion

- Is this more a game of luck or skill?
- Which of the three activities—***Square Off, Four in a Row,*** or ***Cross Over***—offers more opportunities to block the other team? Why?
- Would you prefer to play these games with a partner or without? Why?
- Do you prefer to play offensively or defensively? Which works best for you? Is one strategy more effective? What happens if you both play defensively?
- What strategies have you and your team partner learned from each other?
- ***Four in a Row*** is similar to the games Othello and Pente, in which defense is important. How does the dice toss influence strategy? Is this activity more a game of defense or offense?
- Keep a recording of each dice toss. Which combinations were tossed the most? The least?

Student Directions

How to Play – *Square Off*

- Each team tosses a die. The team with the higher number goes first.
- Team A tosses a die and finds an equation on the chart for which the tossed number is the solution for n. If the solution already has a token on it, the team loses a turn.
- Team A places a token on the equation. If Team B challenges the solution and the solution is incorrect, Team A's **turn is over.** If Team B challenges Team A's solution and the solution is correct, Team B **loses its next turn.**
- Taking turns, the two teams attempt to arrange four tokens to form any size square on the chart (2-by-2, 3-by-3, 4-by-4, and so on). The orientation of the square can be on the diagonal.
- The first team to form three squares wins.

How to Play – *Cross Over*

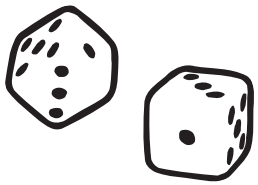
- Each team tosses a die. The team with the higher number goes first.
- Team A tosses a die and finds an equation on the chart for which the tossed number is the solution for n. If the solution already has a token on it, the team loses a turn.
- Team A places a token on the equation. If Team B challenges the solution and the solution is incorrect, Team A's **turn is over.** If Team B challenges Team A's solution and the solution is correct, Team B **loses its next turn.**
- With each toss of the die, teams attempt to place their tokens so they form a continuous path zigzagging vertically, horizontally, or diagonally from one side of the chart to the other.
- The first team to form a continuous path connecting both sides wins.

Variation:

- Team A starts play in either the outside right or outside left column of the chart. If no box in either column contains the solution to the team's first die toss, the team loses a turn. If Team A does place a token in either outside column on the first toss, Team B must place its token in the *opposite* outside column. A token may not be placed on an occupied box.

How to Play – *Four in a Row*

- Each team tosses a die. The team with the higher number goes first.
- Team A tosses a die and finds an equation on the chart for which the tossed number is the solution for n. If the solution already has a token on it, the team loses a turn.
- Team A places a token on the equation. If Team B challenges the solution and the solution is incorrect, Team A's **turn is over.** If Team B challenges Team A's solution and the solution is correct, Team B **loses its next turn.**
- Each team attempts to line up four tokens, vertically, horizontally, or diagonally before the opposing team does.
- The first team to align four tokens in a row wins.



Solve for *n* – Chart 1

$7 \div n = 7 \cdot 1$ $n =$	$60 \div n = 30$ $n =$	$21 \div n = 7$ $n =$	$7 + (-2) = n$ $n =$	$60 \div 15 = n$ $n =$
$8 - n = 6$ $n =$	$(4 \cdot 4) - 10 = n$ $n =$	$9 - n = 4$ $n =$	$-5 + 6 = n$ $n =$	$5 + n = 3 \times 3$ $n =$
$(n \cdot 3) + 4 = 19$ n =	$33 \div n = 11$ $n =$	$2 + n + 5 = 11$ $n =$	$\frac{1}{8} \cdot 48 = n$ $n =$	$n \cdot n = 9$ $n =$
$42 \div n = 7$ $n =$	$n \cdot n \cdot n \cdot n = 1$ $n =$	$(8 + n) - 2 = 11$ $n =$	$32 \div n = 8$ $n =$	$50 \div 25 = n$ $n =$
$\frac{1}{5} \cdot 20 = n$ $n =$	$24 \div n = 8$ $n =$	$\frac{2}{3} \div \frac{4}{6} = n$ $n =$	$44 \div 22 = n$ $n =$	$54 \div n = 9$ $n =$

Directions for Table Completion Activities

Objectives

- Recognize mathematical patterns.
- Practice describing number patterns as rules or algebraic expressions.
- Employ mathematical reasoning.
- Transition from thinking arithmetically to thinking algebraically.

Prerequisite

Students complete the ***Writing Rules*** lessons.

Introduce the ***Table Completion*** activities by demonstrating on an overhead or interactive whiteboard and playing against the class. Two teams with two students on a team are suggested. Teams give students an opportunity to discuss moves and strategies and provide a check on correct computation.

Materials

- Table Completion chart for each pair of teams
- Dice
- Calculator

How to Play

- The two teams look at each table on their shared chart and agree on what operation or function was performed to arrive at the given answers. Each team records the rule for its table. The teams will use those functions during the activity to complete their respective tables.
- Each team tosses a die. The team with the higher number goes first.
- Taking turns, the teams toss 2 dice and find the sum. The sum = n. The team whose turn it is applies the rule to n and records the value in the box next to the sum.
- If the sum has already been played, the team loses a turn.
- If the team records the wrong number, the team erases the number and loses that turn.
- The first team to complete its chart wins.

Suggestion

A calculator is suggested for activities 6, 8, 9, and 10. If students are struggling, suggest that they use a calculator with all the activities.

Discussion

- Is this a game of luck or skill or both?

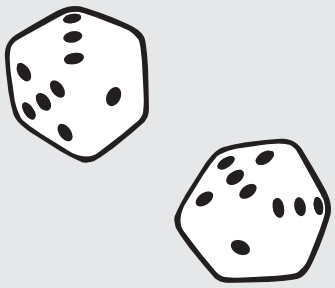
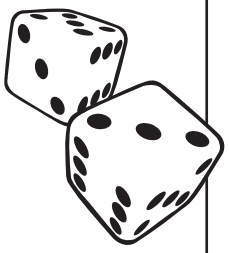


Table Completion 1

How to Play

- Each team tosses a die. Higher number goes first.
- Look at the two tables. What function was performed on the numbers in the "Sum" column to get the answers shown? If both teams agree on the function, record it in the box labeled "Rule." Then use the function to complete your team's table.
- Toss 2 dice. Find the sum. Perform the function and record the solution next to the sum in the table.
- If the sum has already been tossed, lose a turn.
- The first team to complete their table wins.



Team: _____

Sum	Solution
Rule:	
2	
7	
12	24
3	
5	
11	
6	
8	
4	
9	
10	
100	

Team: _____

Sum	Solution
Rule:	
3	
6	
10	
9	
12	
4	
11	
5	
7	
2	4
8	
100	200